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Renyi Type Holographic Dark Energy

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Abstract — Dark energy is one of the prominent mysteries of the universe that still awaits a solution. One of the plausible ways to collect data about any formation or understand its information capacity is to investigate the entropy of that formation. In this study, Renyi Holographic Dark Energy (RHDE) matter distribution is analyzed within the framework of General Relativity Theory, considering homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time. Hubble parameter and RHDE density were used to obtain exact solutions of Einstein field equations. The analysis of the obtained solutions was performed by drawing evolution graphs for redshift z.

Keywords *Renyi holographic dark energy, FRW universe model, Hubble parameter*

Mathematics Subject Classification (2020) 83C05, 83C15

1. Introduction

Observations suggest that the universe is accelerating in two phases: the very early inflationary phase and the current phase. An exotic component with a large negative pressure, called dark energy (DE), which accounts for about 70% of the universe's energy density, is thought to cause the accelerated expansion [1]. In addition, the second largest component of our universe is dark matter (DM). The origin and nature of dark matter and dark energy are not fully known. Different theoretical models have been constructed to explain and interpret the accelerating universe.

The cosmological constant lambda cold dark matter (ΛCDM) is proposed as the simplest dark energy model. Although ΛCDM is consistent with current observations, it suffers from the problem of coincidence and fine-tuning. [2]. For this reason, we are trying to investigate the origin of dark energy based on the holographic principle [3]. With the introduction of the holographic principle into cosmology, holographic dark energy (HDE) was proposed [4]. Holographic dark energy is based on the use of various horizons as the radius of the universe. It can explain the current acceleration and is supported by many observations [5, 6], making holographic dark energy an interesting model. Recently, dark energy models, such as Tsallis holographic dark energy, Renyi holographic dark energy (RHDE), and Sharma Mitall holographic dark energy, have been proposed to investigate cosmological phenomena using extended entropy formalisms, such as Tsallis [7], Renyi [8], and Sharma Mitall [9]. The energy density of RHDE is as follows:

$$\rho_R = \frac{3c^2}{8\pi L^2}\left(1+\pi\delta L^2\right)^{-1} \tag{1.1}$$

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where δ is the true non-extensibility (NE) parameter measuring the degree of non-extensibility, L is the IR cut-off, and c is a numerical constant [10,11]. If the Hubble horizon is taken as the IR cut-off $\left(L = \frac{1}{H}\right)$, then (1.1) becomes the following equation, and the energy density of the RHDE is obtained as follows [10]:

$$\rho_R = \frac{3c^2H^2}{8\pi\left(\frac{\pi\delta}{H^2}+1\right)} \tag{1.2}$$

There are many studies on RHDE. Some of these studies can be summarized as follows. Prasanthi and Aditya [1] have studied the RHDE model for the Bianchi VI_0 universe in the General Relativity Theory. Bhattacharjee [12] has investigated the dynamics of Tsallis and Renyi holographic dark energy in Friedmann-Robertson-Walker (FRW) space-time using a hybrid scale factor. Dubey et al. [10] have analyzed RHDE model in a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric using different values of the parameter δ. Dixit et al. [13] have investigated the behavior of RHDE in the flat FRW universe in the framework of $f(R,T)$ gravity using the Granda-Oliveros and Hubble horizons. Sharma and Dubey [14] have obtained a solution using the RHDE deceleration parameter in the flat FRW universe. They have also calculated state finder parameters to understand the geometrical behavior of the model using observational data. Moreover, in 2020, Sharma and Dubey [15] studied the RHDE model for the FRW universe in Brans-Dicke cosmology. Saha et al. [16] have investigated the interacting and non-interacting RHDE models in the Dvali-Gabadadze-Porratidal braneworld framework. In addition, Saha et al. [17] have researched the distribution of matter with Barrow holographic dark energy and viscous fluid in the form of pressure-less dark matter using different scale factors for the flat FRW universe. Liu et al. [18] have analyzed the quintessential dark energy of the Kerr black hole by testing it through observational data using quasi-periodic oscillations. Ranjit et al. [19] have studied models of the universe with interacting Tsallis holographic dark energy in the Chern-Simons alternative gravitational theory. Yılmaz and Güdekli [20] have investigated FLRW universe models with modified Chaplygin gas and cosmological constants, one of the dark energy candidates. Koussour et al. [21] have proposed a new equation of state parameter for dark energy in the $f(Q)$ alternative theory of gravitation. Koussour et al. [22] have obtained solutions for various Hubble parameters in scalar field dark energy models.

The outline of the article is as follows: In section 2, the General Relativity Theory formulation is provided, and the field equations for the RHDE model are obtained using the FRW metric. In section 3, solutions to Einstein field equations are provided. Moreover, in this section, the solutions are analyzed with the help of graphics. Finally, in the section 4, the planned future studies are mentioned.

2. Field Equations

The General Relativity theory attempts to explain the universe's structure on a large scale. The field equations in this theory are expressed as follows:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu} \tag{2.1}$$

Here, $8\pi G = c^4 = 1$ can be taken. The general form of the homogeneous and isotropic FRW metric in spherical coordinates (r,θ,ϕ,t) is as follows:

$$ds^2 = dt^2 - A^2\left(\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{2.2}$$

Here, it can take the values $\kappa = -1$, $\kappa = 0$, or $\kappa = 1$. If $\kappa = -1$, then it refers to the open universe model, if $\kappa = 0$, then it refers to the flat universe model, and if $\kappa = 1$, then it refers to the closed universe model. The universe is assumed to be filled with matter and a fluid known as holographic dark energy. The energy-momentum tensor for matter is defined as follows:

$$T^{m}_{\mu\nu} = \rho_m u_\mu u_\nu$$

where ρ_m shows the density of matter [1]. The energy-momentum tensor ($T^{R}_{\mu\nu}$) of RHDE is as follows:

$$T^{R}_{\mu\nu} = (\rho_R + p_R) u_\mu u_\nu - p_R\, g_{\mu\nu}$$

where p_R and ρ_R denote RHDE pressure and energy density, respectively [1]. The total energy-momentum tensor is expressed as follows:

$$\begin{aligned} T_{\mu\nu} &= T^{m}_{\mu\nu} + T^{R}_{\mu\nu} \\ &= (\rho_m + \rho_R + p_R) u_\mu u_\nu - p_R g_{\mu\nu} \end{aligned} \tag{2.3}$$

We obtain the Einstein field equations from (2.1)-(2.3) as follows:

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\kappa}{A^2} = -p_R \tag{2.4}$$

and

$$\frac{3\dot{A}^2}{A^2} + \frac{3\kappa}{A^2} = \rho_m + \rho_R \tag{2.5}$$

Here, the dot shows a derivative concerning cosmic time t.

3. Results and Discussions

As can be observed from (2.4) and (2.5), there are two equations with four unknowns as A, ρ_m, ρ_R, and p_R. We need two additional equations to solve this equations system.

i. To obtain a solution, we can first use the Hubble parameter. The Hubble parameter, a cosmological parameter that expresses the universe's expansion rate, defines the universe's expansion rate by a numerical value, also called the Hubble constant (H_0). This parameter helps us understand the universe's expansion rate and obtain some critical information about the universe's past. In this study, the Hubble parameter suggested by Pacif et al. [23] is taken:

$$H = \frac{\dot{a}}{a} = \frac{\beta}{\sqrt{t+\alpha}} \tag{3.1}$$

Here, a denotes scale factor and α and β are real constants.

ii. As the second equation, by estimating the Hubble horizon as an IR cut-off, we can take the energy density of the RHDE as in (1.2).

From (3.1), the metric potential (is also equal to the scale factor a) A is

$$A = c_1 e^{2\sqrt{t+\alpha}\,\beta} \tag{3.2}$$

Here, c_1 is an integral constant. From (1.2) and (3.1), we obtain energy density of RHDE as

$$\rho_R = \frac{3\beta^4 c^2}{8\left(\delta\left(t+\alpha\right)\pi + \beta^2\right)\left(t+\alpha\right)\pi} \tag{3.3}$$

Furthermore, from (2.4), (2.5), (3.2), and (3.3), the energy density of matter and pressure are obtained as follows:

$$\rho_m = \frac{3e^{-4\sqrt{t+\alpha}\,\beta}\kappa}{{c_1}^2} - \frac{3\beta^2\left(-8\delta\left(t+\alpha\right)\pi^2 - 8\beta^2\pi + c^2\beta^2\right)}{8\left(\delta\left(t+\alpha\right)\pi + \beta^2\right)\left(t+\alpha\right)\pi}$$

and

$$p_R = -\frac{e^{-4\sqrt{t+\alpha}\,\beta}\kappa}{{c_1}^2} - \frac{3\left(\sqrt{t+\alpha}\,\beta - \frac{1}{3}\right)\beta}{\left(t+\alpha\right)^{\frac{3}{2}}} \tag{3.4}$$

Finally, we can determine the equation of state (EoS) parameter by using the formulation $\omega = \frac{p_R}{\rho_R}$ where ω denotes the EoS parameter:

$$\omega = -\frac{8\pi\left(3\beta\sqrt{t+\alpha}-1\right)\left(\delta\left(t+\alpha\right)+\beta^2\right)}{3\,\beta^3c^2\sqrt{t+\alpha}} - \frac{8\pi\kappa\left(\delta\left(t+\alpha\right)\pi+\beta^2\right)\left(t+\alpha\right)}{3{c_1}^2c^2\beta^4e^{4\beta\sqrt{t+\alpha}}}$$

When the solutions are investigated, it is observed that there is a singularity at $t = -\alpha$. However, since the constant α is positive and t can never have a negative value, $t = -\alpha$ has no problems for the solutions. In order to draw and analyze the graphs of the physical variables we obtain in the solutions, we need the values of the constants in the solutions. We can get the values of the constants in the solutions by using some observational values. Table 1 contains the values of the constants we use to draw the graphs. When obtaining the values in Table 1, $t_0 = 13.8$ Gyr was taken.

Table 1. Values of constants

Data Set	α	β	c_1	δ	c
SN Ia [24]	1.0	0.4813	0.0246	1.4	70
SN Ia + H(z) + BAO/CMB [25]	1.6	0.2770	0.1137	1.4	110
SN Ia + BAO +H(z) [26]	1.3	0.3762	0.0537	1.4	90
CC+SN Ia+BAO+R18 [27]	1.6	0.2963	0.0977	1.4	120

Using the values in Table 1, a graph of the variation in metric potential over time was drawn for four different observation values. When Figure 1 is analyzed, it is observed that the metric potential exhibits similar behavior for all four observation values up to a certain point and increases over time. Still, after a certain point, the increase accelerates for SN Ia. The fact that the metric potential increases over time within four different observation values shows that the universe model has an expansion.

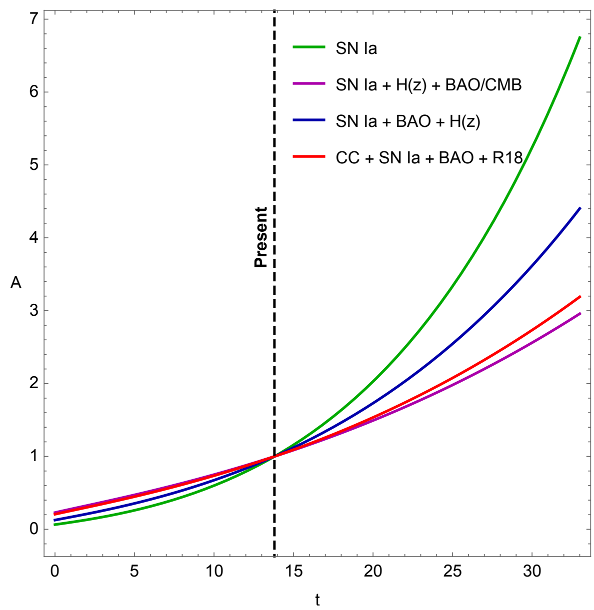

Figure 1. Variation of metric potential versus cosmic time t

The following equation expresses the relationship between redshift and scale factor:

$$1 + z = \frac{a(t_0)}{a(t)} \tag{3.5}$$

where $a(t_0)$ is the present value of the scale factor. In this study, $a(t_0) = 1$ is assumed. From (3.1) and (3.5), a relationship between the redshift variable z and time can be obtained as follows:

$$t = t_0 - \frac{\sqrt{t_0 + \alpha}\,\ln(1+z)}{\beta} + \frac{\ln(1+z)^2}{4\beta^2} \tag{3.6}$$

With the help of (3.3) and (3.6) and using the values in Table 1, the variation graph of the energy density of RHDE according to redshift is presented in Figure 2. The energy density is expected to increase in dark energy models according to z. When Figure 2 is investigated, an increase is observed for all four observation values. However, the increase is faster for the observation values SN Ia + H(z) + BAO/CMB and CC+SN Ia+BAO+R18.

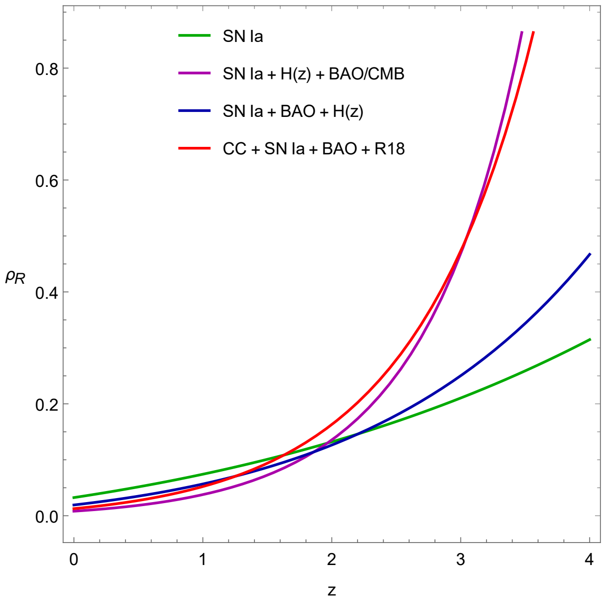

Figure 2. Variation of the energy density of RHDE versus redshift.

Similarly, with the help of (3.4) and (3.6) and using the values in Table 1, the variation graph of the energy density of RHDE according to redshift is provided in Figure 3. In dark energy models, the pressure is expected to decrease concerning z. When Figure 3 is investigated, a decrease is observed for all four observation values. However, it can be observed from Figure 3 that the observation values of SN Ia + H(z) + BAO/CMB and CC + SN Ia + BAO + R18 start to increase after a certain point.

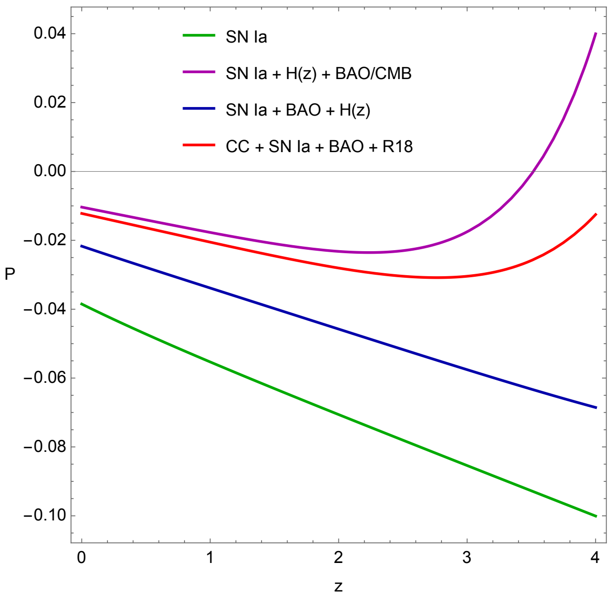

Figure 3. Variation of pressure versus redshift

Figure 4 shows the variation of the equation of state parameter according to redshift. When Figure 4 is investigated, the equation of state parameter behaves similarly in four different observation values.

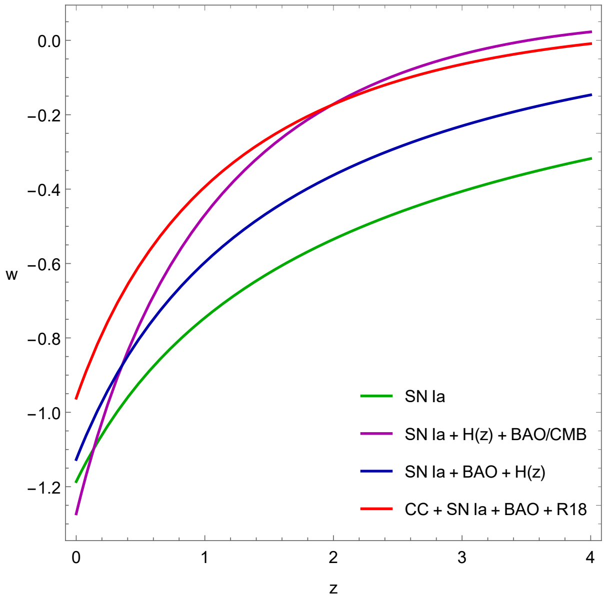

Figure 4. Variation of equation of state versus redshift

4. Conclusion

This article has investigated the behavior of the RHDE matter distribution in the homogeneous and isotropic FRW universe model, which best describes today's universe in the framework of General Relativity Theory. In future studies, it will be worthwhile to investigate universe models with RHDE matter distribution within the framework of Lyra theory, $f(R)$ theory, and $f(Q)$ theory.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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